

THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH1010G/H University Mathematics 2014-2015
Assignment 2

- Due date: 12 Feb, 2015 (before 17:00)
- Remember to write down your name and student number
- Please work on ALL questions below.

Questions from Thomas Calculus:

Exercise 2.6: 35, 36, 83

Exercise 3.6: 37, 45, 55

1. Let $f(x) = x|x|$.
 - (a) Find $f'(x)$ for $x > 0$ and $x < 0$ respectively.
 - (b) Prove that $f'(0)$ exists.
 - (c) Prove that $f'(x)$ is continuous at $x = 0$.
(Caution: $f(x)$ is continuous at $x = 0$ by (b), but we do not know whether $f'(x)$ is also continuous at $x = 0$.)
2. By using Mean Value Theorem, prove that $|\sin x - \sin y| \leq |x - y|$ for any real numbers x and y .
(Hint: consider the cases $x < y$, $x = y$ and $x > y$.)
3. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function such that
 - (i) $f(x + y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$.
 - (ii) $f(x) = 1 + xg(x)$, where $\lim_{x \rightarrow 0} g(x) = 1$.Show that $f'(x) = f(x)$ for all $x \in \mathbb{R}$.